

CALCULUS

When taught in high school, calculus should be presented with the same level of depth and rigor as entry-level college and university calculus courses. These standards outline the essential elements of the College Board syllabus for the Calculus AB section of the Advanced Placement Examination in Mathematics. It is possible to extend this curriculum to the BC level of the Advanced Placement Exam for classes that have sufficient time to do so. However, much of this curriculum is already well beyond the requirements of the PA Standards.

1.0 Limits:

PA Standards

Students define and evaluate limits using technology to verify results.

- 1.1 Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, limits of integer-valued functions, limits of piecewise functions, and limits involving infinity.
- 1.2 Know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity.
- 1.3 Use theorems evaluating the limits of sums, products, quotients, and composition of functions.
- 1.4 Use graphing calculators to verify and estimate limits. 2.2.11.F
- 1.5 Find limits of undefined and indeterminate expressions by using a variety of algebraic and trigonometric techniques.
- 1.6 Use special limits, such as the limits of $(\sin(x))/x$ and $(1-\cos(x))/x$ as x tends to zero or infinity.

2.0 Differential Calculus:

Students demonstrate an understanding of differentiation by solving contextual problems.

- 2.1 Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
- 2.2 Demonstrate an understanding of and the application of the intermediate value theorem and the extreme value theorem.

- 2.3 Generalize the concept of slope by investigating the difference quotient of a function and finding its limit as h approaches zero.
- 2.4 Demonstrate an understanding of the formal definition of the derivative of a function at a point and the idea of differentiability.
- 2.5 Understand that the derivative of a function is its slope formula. That is, the derivative is an expression which yields the value of the slope of the tangent line to the graph of the function for any given point of the function.
- 2.6 Know the Power, Product, Quotient, and Chain Rules for differentiation and use them to determine the derivatives of algebraic, exponential, logarithmic, and trigonometric functions.
- 2.7 Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth, that involve the rate of change of a function. 2.4.11.E;
2.5.11.A
- 2.8 Understand the relation between differentiability and continuity.
- 2.9 Compute derivatives of higher orders.
- 2.10 Understand the difference between finding derivatives explicitly and implicitly.
- 2.11 Use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth. 2.4.11.E;
2.5.11.E
- 2.12 Use differentiation to sketch, by hand, graphs of functions. Identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing or where the function is concave up or concave down. 2.11.11.A, B, C
- 2.13 Use differentiation to solve optimization (maximum-minimum) problems in a variety of pure and applied contexts. 2.4.11.E;
2.11.11.A, B
- 2.14 Use differentiation to solve related rate problems in a variety of pure and applied contexts. 2.4.11.E
- 2.15 Know and apply Rolle's theorem, the mean value theorem for derivatives, and L'Hopital's rule.
- 2.16 Know Newton's method for approximating the zeros of a function. (This is an ideal example of a reiterative algorithm.) 2.11.11.D

3.0 Integral Calculus:

PA Standards

Students demonstrate their understanding of integration by solving contextual problems.

- 3.1 Investigate the summation symbol: $\sum_{i=1}^n x_a$
- 3.2 Compute, by hand, the integrals of a variety of functions including algebraic, trigonometric, and exponential, by using techniques of integration, such as substitution.
- 3.3 Understand the difference between the definite and the indefinite integral.
- 3.4 Know the definition of the definite integral by using Riemann sums. Use this definition to approximate integrals. 2.11.11.D
- 3.5 Apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals. 2.4.11.E
- 3.6 Demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.
- 3.7 Use definite integrals in problems involving area, velocity, acceleration, volume of a solid (disk, washer, and shell methods), area of a surface of revolution, length of a curve, and work. 2.11.11.E
- 3.8 Find the volume of a solid using different axes of revolution.
- 3.9 Find the volume of a solid with known cross-sections.
- 3.10 Understand the algorithms involved in Simpson's rule, the Trapezoid Rule and Newton's method. Use calculators or computers or both to approximate integrals numerically. 2.2.11.F;
2.11.11.D

4.0 Exponential and Logarithmic Functions:

Students practice differentiation and integration on a variety of functions and interpret results in problem solving.

- 4.1 Review the definition and graph of the natural log function. 2.8.11.S
- 4.2 Review the definition and graph of the exponential function. 2.8.11.S
- 4.3 Differentiate and integrate exponential functions.

PA Standards

- 4.4 Use analytical skills to investigate exponential growth and decay. 2.11.11.C
- 4.5 Differentiate and integrate logarithmic expressions.

5.0 Extensions in the BC level of the Advanced Placement Examination in Calculus:

- 5.1 Explore the process of integration by parts.
- 5.2 Investigate partial fractions.