

# TRIGONOMETRY

Trigonometry uses the theory and techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined algebraically and geometrically. Familiarity with these functions as well as the ability to employ basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

---

## 1.0 Trigonometric Functions and Graphs:

## PA Standards

Students measure angles in degrees and radians. They know the definitions and can interpret the graphs of the trigonometric functions. They can graph the sum of two functions.

- |     |  |                                      |
|-----|--|--------------------------------------|
| 1.1 | Understand the concept of an angle and how to express its measure.   | 2.3.11B                              |
| 1.2 | Convert between degrees and radians.   |                                      |
| 1.3 | Understand the concept of arc length and know how to calculate it.   |                                      |
| 1.4 | Use radian measure to investigate arc length, angular velocity, and linear velocity.   |                                      |
| 1.5 | Define and graph periodic functions and use scientific and/or graphing calculators to explore these relationships.   | 2.8.11.Q;<br>2.10.11.A;<br>2.11.11.A |
| 1.6 | Define and understand the characteristics of even and odd functions with an emphasis on symmetry considerations.   |                                      |
| 1.7 | Know the definition of $\sin \theta$ and $\cos \theta$ as y- and x-coordinates of points on the unit circle and are familiar with the evaluations and graphs of the sine and cosine functions.                 | 2.10.11A                             |
| 1.8 | Graph functions of the form $f(x) = a \sin b(x - c) + d$ or $f(x) = a \cos b(x - c) + d$ to interpret a, b, c, and d in terms of amplitude, period, phase shift, and vertical shift.                           | 2.10.11A;<br>2.11.11A, B             |
| 1.9 | Know the definitions of the tangent and cotangent functions and evaluate and graph them over any given interval. The functions are in the form $f(x) = a \tan b(x - c) + d$ and $f(x) = a \cot b(x - c) + d$ . | 2.8.11.O, Q, S, T                    |

- 1.10 Know the definitions of the secant and cosecant functions and graph them over any given interval. The functions are in the form  $f(x) = a \sec b(x - c) + d$  and  $f(x) = a \csc b(x - c) + d$ . 2.8.11.O, Q, S, T
- 1.11 Graph the sum of two functions by addition of ordinates.

## 2.0 Right Triangle Trigonometry and Basic Identities:

Students derive and prove trigonometric identities and solve problems involving right triangles such as angle of inclination

- 2.1 Know the identity  $\cos^2(x) + \sin^2(x) = 1$  and its applications. 2.10.11B
- 2.2 Know that the identity  $\cos^2(x) + \sin^2(x) = 1$  is equivalent to the Pythagorean Theorem (i.e., students can prove this identity by using the Pythagorean Theorem and, conversely, they can prove the Pythagorean Theorem as a consequence of this identity). 2.4.11A  
2.4.11B
- 2.3 Derive and prove other trigonometric identities and simplify them by using the identity  $\cos^2(x) + \sin^2(x) = 1$ . For example, use this identity to prove that  $\sec^2(x) = \tan^2(x) + 1$ .
- 2.4 Know that the tangent of the angle of inclination of a line is equal to the slope of that line.
- 2.5 Use trigonometry to determine unknown sides or angles in right triangles. 2.10.8A

## 3.0 Oblique Triangles:

Students apply the Law of Sines and the Law of Cosines to solve problems. They investigate the ambiguous case for the Law of Sines.

- 3.1 Know the Law of Sines and the Law of Cosines and apply those laws to solve problems involving acute and obtuse triangles.
- 3.2 Understand that the Law of Cosines is a generalization of the Pythagorean Theorem.
- 3.3 Investigate the ambiguous case for the Law of Sines.

- 3.4 Explore the connection between both the Law of Sines and the Law of Cosines and the congruence criteria from geometry.
- 3.5 Determine the area of a triangle, given the lengths of two sides and the measure of their included angle.

---

**4.0 Trigonometric Identities:**

Students perform manipulations with trigonometric expressions and identities.

- 4.1 Demonstrate an understanding of the trigonometric identities and their proofs and use those identities to prove and/or simplify other trigonometric expressions. 2.4.11.A, B, C
- 4.2 Demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and use those formulas to prove and/or simplify other trigonometric identities. 2.4.11.A, B, C

---

**5.0 Inverse Trigonometric Functions:**

Students know and graph the inverse trigonometric functions. They compute specific values of the trigonometric and inverse trigonometric functions.

- 5.1 Explore the connection between the defined ranges of the inverse functions and certain calculator limitations.
- 5.2 Know the definitions of the inverse trigonometric functions and evaluate and graph the functions in their standard intervals.
- 5.3 Compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions for quadrantal angles and reference angles of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  (i.e.,  $0$  rad,  $\pi/6$  rad,  $\pi/4$  rad,  $\pi/3$  rad,  $\pi/2$  rad, etc.).

---

**6.0 Applications:**

Students understand and model applications of trigonometry in contextual situations.

- 6.1 Use trigonometry in a variety of applications and word problems (e.g., surveying, navigation, determination of inaccessible distances). 2.4.11.E

---